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Keywords

Cointegration, Johansen procedure, Bierens method, robustness, simulation, Hausdorff distance

JEL Classifications

C13, C15, C32

Comments

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Abstract

The paper compares the cointegration methods of Johansen and Bierens by means of simulations and a real world example. Drawing on the fact developed in a companion paper, namely that the Johansen procedure has robustness properties against ARMA systems and the Bierens procedure is designed for ARMA systems, a comparison between the two seems to be a reasonable exercise, to find out in which cases the Bierens procedure outperforms the Johansen procedure.
It turns out that generally the Johansen procedure outperforms the Bierens procedure in terms of the quality of the tests as well as the approximation quality of the estimated cointegrating space to the true cointegrating space.

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1 Introduction

Cointegration has been one of the most prominent topics in econometrics during the last ten years. Numerous papers on theoretical problems as well as empirical applications have been published.
Among all the different methods for estimation and testing for cointegration that are available the method developed by Johansen [7, 8, 9, 11] is the one that is most widely used. The Johansen method starts from a fully parametrized vector autoregressive model (VAR). Therefore, one obtains estimates of the cointegrating vectors, the adjustment parameters, the short-run coefficient matrices and the variance-covariance matrix of the residuals. This allows for a detailed analysis of the dynamic properties of the model at hand, including the testing for (weak) exogeneity of variables. Furthermore one can also try to link the results to economic theory via the construction of a structural vector autoregression (SVAR).
A large variety of hypotheses on the cointegrating space or on individual cointegrating vectors can also be formulated and tested, as can hypotheses on the adjustment parameters.
The complete dynamic specification is on the other hand probably the drawback of this method since one is restricted to analyze VARs only. The restrictions are not as stringent as they might seem at first glance. It can be shown that, if the data generating process is not purely autoregressive but autoregressive moving average one can overcome this problem by adding a sufficient number of lags to an autoregressive approximation of the process and use the Johansen procedure for the autoregressive approximation. To be precise, Saikkonen [16] has shown that, generalizing the work of Said and Dickey [15] to the multivariate case, one can consistently estimate the cointegrating space of a general $I(1)$ process when one increases the lag order of an autoregressive approximation with an appropriate rate for increasing sample size. Another possibility is to use the results of Yap and Reinsel [19], who have derived the maximum likelihood estimator for the cointegrating space in a Gaussian vector autoregressive moving average (VARMA) framework, but due to the small sample sizes that are usually available in macro-econometric applications one is restricted to estimate simple models. If the data generating process is VARMA, but one uses the Johansen procedure for a VAR, Wagner [18] has shown that the Johansen procedure still produces consistent estimates of the cointegrating vectors under some conditions on the moving average part of the model. This result is described in some detail in Section 2.
Despite all the above results it might be the case that one does not necessarily have to specify the dynamics of the model if one is only interested in the cointegrating space. To address this question Bierens [2, 3] has developed a non-parametric procedure to estimate and test for cointegration. The procedure is designed for general processes integrated of order 1, see Section 2 for a discussion. This offers an alternative to the Johansen procedure if one cannot be sure whether the data are indeed generated by a low-order vector autoregression

and one does not want to fully parametrize a VARMA model, e.g. because of short sample sizes or poor statistical properties of estimated low-order autoregressive (moving average) models. Compared to the parametric cointegration approaches by using Bierens' approach fewer results are obtained since one only gets estimates of the cointegrating space. This is the price that has to be paid for increased flexibility of the models that are allowed for as data generating processes, again in comparison to the Johansen procedure. Bierens has also developed tests for hypotheses on the cointegrating vectors.
There are already a couple of papers published that analyze the behavior of the Johansen estimation and testing procedure under misspecification by means of simulation studies (e.g. Bewley and Yang [1], Podivinsky [14] or Toda [17], and also Wagner [18] is investigating the empirical relevance of the above mentioned theoretical results via a simulation study).
The aim of this paper is to explore whether the method developed by Bierens can step in when there are doubts about the VAR nature of the data generating process. The results from Wagner [18] suppose that although the estimates generated by applying the Johansen procedure are consistent for ARMA processes as well, it is the testing step that tends to have low discriminatory power for finding the correct dimension of the cointegrating space. At this stage it can be assumed that the Bierens test could add some extra value in helping to find the correct dimension of the cointegrating space. Since the Bierens procedure is designed implicitly for ARMA processes, it will also be interesting to find out how much moving average dynamics we have to put to an autoregressive model to make the Bierens test superior to the Johansen test. In this interpretation we look at autoregressive moving average processes as disturbed autoregressive processes.
The paper is organized as follows. In Section 2 the methods are briefly described and discussed, in Section 3 the set-up and the results of the simulations performed are presented and discussed and Section 4 gives the results of an empirical application of both methods to see their relative performance on real data. Section 5 concludes.

2 A description of the methods

Johansen [7, 8, 9, 11] has derived the estimation and testing procedure for cointegration analysis in a Gaussian vector autoregressive framework.[1] Since this method is very well known by now, we will keep the discussion very brief. For notational simplicity we ignore deterministic factors like intercepts, trends or seasonal dummies. Let

$$a(L)x_t = \varepsilon_t \tag{2.1}$$

[1] We restrict ourselves here to a discussion of the $I(1)$ case. Johansen [10, 12] has developed estimation and testing procedures for cointegration in VAR systems integrated of order 2.

where x_t is an m-dimensional random variable, $a(z) = I_m - a_1 z - \ldots - a_p z^p$[2] and ε_t is iid $N(0, \Sigma)$. The assumptions concerning $a(z)$ are that det $a(z) = 0$ implies $z = 1$ or $|z| > 1$. So all the roots of the polynomial $a(z)$ are either outside the unit circle or at $z = 1$. As is well known cointegration manifests itself in a reduced rank of the matrix $a(1)$. Let r denote the rank of this matrix, then r is also the dimension of the cointegrating space.
The above model can be re-written in Error Correction Form

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_{p-1} \Delta x_{t-p+1} + \Pi x_{t-p} + \varepsilon_t$$

It is easy to see that $\Gamma_i = -I_m + a_1 + \ldots + a_{i-1}$ for $i = 2, \ldots, p-1$ and $\Pi = -a(1) = \alpha\beta'$.
The columns of the matrix β span the cointegrating space. To guarantee that x_t is not in fact integrated of an order higher than one, a further assumption is required. The matrix

$$\alpha'_\perp (\frac{\partial a(L)}{\partial L}|_{L=1}) \beta_\perp$$

has to have full rank $m - r$. Here for a matrix $P \in \mathbb{R}^{m \times r}$ with full rank the matrix $P_\perp$ is $\in \mathbb{R}^{m \times m-r}$, has full rank and spans the orthogonal complement of the space spanned by the columns of P, i.e. $P'_\perp P = 0$.
The maximum likelihood estimation of β proceeds as follows. In a first step one can concentrate the likelihood function with respect to the parameter matrices $\Gamma_1, \ldots, \Gamma_{p-1}$ by running two OLS regressions. This is achieved by a regression of Δx_t and a regression of x_{t-p} on the lagged differences $\Delta x_{t-1}, \ldots, \Delta x_{t-p+1}$. The residuals of these two regressions are denoted with R_{0t} and R_{pt}. The product moment matrices of these residuals are given by

$$S_{ij} = \frac{1}{T} \sum_{t=1}^{T} R_{it} R'_{jt}, \qquad i, j = 0, p$$

Using the above quantities the maximum likelihood estimates of β are given by the eigenvectors corresponding to the r largest eigenvalues $\hat{\lambda}_1, \ldots, \hat{\lambda}_r$ of

$$|\lambda S_{pp} - S_{p0} S_{00}^{-1} S_{0p}| = 0$$

The likelihood ratio test statistic of $H_0 : dim(\beta) \leq r$ against the alternative $dim(\beta) = m$ is given by

$$-2ln(Q)_\eta = T \sum_{i=r+1}^{m} ln(1 - \hat{\lambda}_i)$$

This test is denoted trace or η test. We can also test the hypothesis $H_0 : dim(\beta) \leq r$ against the alternative $dim(\beta) = r + 1$. This leads to the max or ξ test with test statistic

$$-2ln(Q)_\xi = ln(1 - \hat{\lambda}_{r+1})$$

[2]We use L to refer to the Lag operator, and z refers to a complex valued variable.

The Johansen method thus gives at the same time estimates of the cointegrating vector and test statistics for determining the number of cointegrating vectors. The critical values of the test statistics are tabulated and can be found e.g. in Osterwald-Lenum [13] or Johansen [11].
Within the framework developed by Johansen there is a variety of possibilities to test for hypotheses on the cointegrating space, see e.g. Johansen [11].
Wagner [18] has shown that the Johansen procedure is delivering consistent estimates of the cointegrating space also if the true model is not autoregressive as assumed above, but is given by

$$a(L)x_t = b(L)\varepsilon_t \tag{2.2}$$

with $a(L)$ as above, (a, b) left co-prime and det $b(z) \neq 0$ for $z = 1$.

Theorem 2.1 *The Johansen estimation procedure for cointegrated vector autoregressive models yields under the form of ARMA misspecification discussed above consistent estimates $\hat{\beta}$ for β.*
The estimates $\hat{\Sigma}$ and $\hat{\Pi}$ are generally not consistent and their limits are given by

$$\hat{\Pi} \xrightarrow{P} \Pi + \xi_p \alpha(\alpha'\alpha)^{-1}\Sigma_{\beta\beta}^{-1}\beta'$$

and

$$\begin{aligned}\hat{\Sigma} \xrightarrow{P} \Sigma - \xi_p \alpha(\alpha'\alpha)^{-1}\Sigma_{\beta\beta}^{-1}\beta'\Sigma_{p0} - \Sigma_{0p}\beta\Sigma_{\beta\beta}^{-1}(\alpha'\alpha)^{-1}\alpha'\xi_p' - \\ -\xi_p\alpha(\alpha'\alpha)^{-1}\Sigma_{\beta\beta}^{-1}(\alpha'\alpha)^{-1}\alpha'\xi_p' + \xi_0 + \xi_p'\end{aligned}$$

With

$$\begin{aligned}\xi_0 &= \sum_{n=1}^{q} b_n \Sigma c_n' - \tau_1 \mu_{**}^{-1}\mu_{*0} \\ \xi_p &= \tau_2 - \tau_1\mu_{**}^{-1}\mu_{*0}\end{aligned}$$

and

$$\tau_1 = \left(\sum_{n=1}^{q} b_n \Sigma c_{n-1}', \sum_{n=2}^{q} b_n \Sigma c_{n-2}', \dots\right);$$

$$\tau_2 = \sum_{m=p}^{q} b_m \Sigma d_{m-p}' \Gamma_k'$$

for $q \geq p$ and $\tau_2 = 0$ for $q < p$. The c_n are the coefficients from the Wold representation for Δx_t, i.e. from $\Delta x_t = c(L)\varepsilon_t$, the b_m are the coefficients from the MA polynomial $b(L)$ and $d_{m-p} = -\sum_{i=j+1}^{\infty} c_i$.

A proof of this theorem is given in Wagner [18].

This result is based on the fact that the cointegrating spaces of system (2.1) and all systems (2.2) are identical, which is also shown in Wagner [18]. The last observation becomes clear by looking at the common trends representation of integrated systems of order 1, which is the content of the famous Granger representation theorem, see e.g. Engle and Granger [5]. To make the argument visible we write the ARMA system as follows:

$$a(L)x_t = u_t$$

$$u_t = b(L)\varepsilon_t$$

which reduces to the AR case if $b(L) = I$. Now the Granger representation theorem derives an MA representation of the above 'autoregressive' representation, which is given by

$$x_t = \beta_\perp(\alpha'_\perp a_1(1)\beta_\perp)^{-1}\alpha'_\perp \sum_{t=1}^{T} u_t + c_1(L)u_t \tag{2.3}$$

Furthermore, $a_1(1)$ is given from $a(L) = a(1) + (1-L)a_1(L)$. The same derivations apply to $c_1(L)$, where $c(L)$ is the inverse of $a(L)/(1-L)$.
Now replace u_t by $b(L)\varepsilon_t$ in (2.3)

$$\begin{aligned} x_t &= \beta_\perp(\alpha'_\perp a_1(1)\beta_\perp)^{-1}\alpha'_\perp \sum_{t=1}^{T}\sum_{j=0}^{q} b_j\varepsilon_{t-j} + c_1(L)b(L)\varepsilon_t \\ &= \beta_\perp(\alpha'_\perp a_1(1)\beta_\perp)^{-1}\alpha'_\perp b(1) \sum_{t=1}^{T}\varepsilon_t + c_1(L)b(L)\varepsilon_t \end{aligned}$$

For this being a common trends representation we need that the second term on the right-hand side of the above equation is stationary.
From the assumptions on $a(L)$ we know that $c_1(L)$ has all its roots outside the unit circle. Therefore, we have to require that also $b(1)$ has no unit roots, to guarantee stationarity of that component. Now, if $b(1)$ is non-singular, we see that the common trends in the first component, in the pure autoregressive case given by $\alpha'_\perp \sum_{t=1}^{T} \epsilon_t$, are subject to a coordinate transformation due to pre-multiplication by $b(1)$ and are now given by $\alpha'_\perp b(1)\sum_{t=1}^{T}\varepsilon_t$.
The above theorem shows that the regularity of $b(1)$ is sufficient, together with the assumption of left co-primeness of $a(L)$ and $b(L)$, so that the cointegrating space of order 1 remains unchanged.
We have seen above that the common trends are given by $\alpha'_\perp b(1)\sum_{t=1}^{T}\epsilon_t$, therefore it is no surprise that the estimate of α is inconsistent since the loading matrix under misspecification is influenced by the MA polynomial. The same holds for the variance matrix of the residuals.
It might be an interesting question to relate these results to the literature on AR estimation of ARMA processes in the stationary case.
Wagner [18] also discusses the fact that regardless of the misspecification the

first r eigenvalues of the generalized eigenvalue problem are converging towards non-zero constants, while the latter go to zero as $O_p(\frac{1}{T})$. This directly implies that the asymptotic power of the *trace* test against the alternative that there are $r+s$ cointegrating vectors is tending to 1, because then the test statistic $-T\sum_{i=r+1}^{m}\ln(1-\hat{\lambda}_i)$ contains s terms that are diverging.[3]

An important remark that has to be made here is (this can be easily deduced from the above) that for integrated processes of any order, the cointegrating spaces of all orders are invariant to MA polynomials $b(L)$ as long as *det* $b(1)\neq 0$. For a discussion of these issues see Deistler and Wagner [4].

The method developed by Bierens on the other hand is designed for Gaussian vector autoregressive moving average processes. A precise description including all the proofs can be found in Bierens [2, 3].
The starting point of Bierens' considerations is the following representation of the integrated vector valued time series

$$x_t = x_{t-1} + u_t$$

where u_t is an m-dimensional stationary process. x_t is assumed to be observed for $t=0,1,\ldots,T$. Under some regularity conditions we can write

$$u_t = C(L)\varepsilon_t$$

where ε_t is white noise with unit variance. The assumption concerning $C(L)$ is that it can be written as

$$C(L) = C_1^{-1}(L)C_2(L)$$

where $C_1(L)$ and $C_2(L)$ are finite order lag polynomials and $\det(C_1(z))$ has all its roots outside the unit circle. $C(L)$ is a $m\times m$ lag polynomial. By construction $C(L)-C(1)$ is zero in each entry, so we can write

$$\begin{aligned}
C(L)\varepsilon_t &= C(1)\varepsilon_t + (C(L)-C(1))\varepsilon_t \\
&= C(1)\varepsilon_t + \frac{C(L)-C(1)}{1-L}(1-L)\varepsilon_t \\
&= C(1)\varepsilon_t + C^*(L)(1-L)\varepsilon_t
\end{aligned} \tag{2.4}$$

with $C^*(L) = \frac{C(L)-C(1)}{1-L}(1-L) = \sum_{j=0}^{\infty} C_j^* L^j$
Denoting by $w_t = C^*(L)\varepsilon_t$ one can write

$$u_t = C(1)\varepsilon_t + w_t - w_{t-1}$$

and

$$x_t = x_0 - w_0 + w_t + C(1)\sum_{j=1}^{t}\varepsilon_j$$

[3] Analogous reasoning works also for the max test.

Under the presence of cointegration, as we have seen before, $C(1)$ will be singular. The vectors spanning the left-kernel of $C(1)$ span the cointegrating space. Furthermore, to exclude cointegration of higher order, the matrix

$$B_r'C^*(1)C^*(1)'B_r$$

has to be regular, where B_r is a matrix spanning the r-dimensional cointegrating space.[4]

The idea behind the non-parametric cointegration approach is to exploit differences in the convergence behavior of certain weighted means of x_t and of Δx_t that occur under the presence of cointegration or otherwise. The means are given by,

$$M_T^x(F) = \frac{1}{T}\sum_{t=1}^{T} F(\frac{t}{T})x_t$$

$$M_T^{\Delta x}(F) = \frac{1}{T}\sum_{t=1}^{T} F(\frac{t}{T})\Delta x_t$$

where F is a continuously differentiable function on the unit interval.

Under the stated assumptions it can be shown (see Lemma 1 in Bierens [3]) that

$$\begin{pmatrix} \frac{M_T^x(F)}{\sqrt{T}} \\ M_T^{\Delta x}(F)\sqrt{T} \end{pmatrix} \rightarrow \begin{pmatrix} C(1)Y_F\sqrt{\int\int F(y)F(z)min(y,z)dydz} \\ C(1)Z_F\sqrt{\int F(y)^2dy} \end{pmatrix} \tag{2.5}$$

where Y_F and Z_F are independent m-variate normally distributed random vectors[5] depending on F as follows

$$Y_F = \frac{\int F(y)W(y)dy}{\sqrt{\int\int F(y)F(z)min(y,z)dydz}}$$

$$Z_F = \frac{F(1)W(1) - \int f(y)W(y)dy}{\sqrt{\int F(y)^2dy}}$$

where $W(.)$ is an m-dimensional Wiener process and $f(y)$ is the first derivative of $F(y)$ with respect to y.

In the case of cointegration, where $C(1)$ is singular, the limiting distribution of (2.5) is singular.

For any cointegrating vector β it follows that $\beta'\frac{M_T^x(F)}{\sqrt{T}} \rightarrow 0$ and $\beta' M_T^{\Delta x}(F)\sqrt{T} \rightarrow 0$. Thus, the convergence rates are different for the weighted means whether they

[4]Note that B_r the matrix that spans the cointegrating space is equivalently the matrix of eigenvectors to the eigenvalue 0 of the matrix $C(1)C(1)'$.

[5]To achieve independence of Y_F and Z_F the function F has to be chosen to satisfy $\int F(y)dy = 0$. All the integrals in this paper are from 0 to 1.

are premultiplied with cointegrating vectors or not. It is this difference that Bierens exploits in constructing his test. The limiting behavior of the weighted means premultiplied with the matrix $B = (\beta_1, \ldots, \beta_r)$ spanning the cointegrating space is given by(see Lemma 2 in Bierens [3]),

$$\begin{pmatrix} B' M_T^x(F)\sqrt{T} \\ B' M_T^{\Delta x}(F) T \end{pmatrix} \to \begin{pmatrix} B' C^*(1) Z_F \int F(y)^2 dy \\ F(1) B' \bar{C}^* Z \end{pmatrix} \qquad (2.6)$$

Z_F is as above, Z is an m-dimensional standard normally distributed random variable and $\bar{C}^* = (\sum_{j=0}^{T} C_j^* C_j^{*\prime})^{\frac{1}{2}}$. These differences can now be used to construct the nonparametric cointegration test.[6] The computation of the test statistic involves choosing a sequence of functions $F_k(y)$, all satisfying $\int F_k(y)dy = 0$, and the construction random matrices involving weighted means of x_t, Δx_t and the functions F_k. These weighted means are then the input in a generalized eigenvalue problem, whereas in the Johansen approach the test statistics are given by the solutions of that eigenvalue problem. An optimal choice for the functions F_k is given by

$$F_k(y) = cos(2k\pi y)$$

which maximize a lower bound of the power function of the test.[7] The abovementioned random matrices are computed as follows

$$\hat{A}_h = \frac{8\pi^2}{T} \sum_{j=1}^{h} j^2 (\frac{1}{T} \sum_{t=1}^{T} cos(2j\pi(t-0.5)/T) x_t)(\frac{1}{T} \sum_{t=1}^{T} cos(2j\pi(t-0.5)/T) x_t)'$$

$$\hat{B}_h = 2T \sum_{j=1}^{h} j^2 (\frac{1}{T} \sum_{t=1}^{T} cos(2j\pi(t-0.5)/T) \Delta x_t)(\frac{1}{T} \sum_{t=1}^{T} cos(2j\pi(t-0.5)/T) \Delta x_t)'$$

Based on the random matrices described above the following can be shown(Bierens [3], Theorem 1 on page 387)

Theorem 2.2 *Let* $(\hat{\lambda}_{1,h}, \ldots, \hat{\lambda}_{m,h})$ *be the ordered solutions of the generalized eigenvalue problem*

$$det(\hat{A}_h - \lambda(\hat{B}_h + \frac{1}{T^2}\hat{A}_h^{-1})) = 0$$

and let $(\lambda_{1,h}, \ldots, \lambda_{m-r,h})$ *be the ordered solution of the generalized eigenvalue problem*

$$det(\sum_{s=1}^{h} X_s^* X_s^{*\prime} - \lambda \sum_{s=1}^{h} Y_s^* Y_s^{*\prime}) = 0$$

[6]The details are given in Bierens [2, 3].

[7]To make the test invariant for drift terms one can choose $F_k(y) = cos(2k\pi(y - \frac{1}{2T}))$.

where the X_i^*'s and the Y_j^*'s are iid $N(0, I_{m-r})$. If x_t is cointegrated with r linear independent cointegrating vectors, then $(\hat{\lambda}_{1,h}, \ldots \hat{\lambda}_{m,h})$ converge in distribution to $(\lambda_{1,h}, \ldots, \lambda_{m-r,h}, 0, \ldots, 0)$.

This theorem is the basis for using $\hat{\lambda}_{m-r,h}$ as a test statistic for testing the null hypothesis that the dimension of the cointegrating space is $r+1$.[8]
The power of the test depends upon, as already mentioned, the choice of the functions F_k and upon the choice of the summation index h. Bierens [2] derives an optimal value for maximizing a lower bound of the power function given the choice of $F(y) = cos(2k\pi y)$. It turns out that in most cases $h = m$ is optimal.[9]
After having decided about the dimension of the cointegrating space, a basis of this space can be estimated as follows. Given that there are r linear independent cointegrating vectors $\beta_1, \ldots, \beta_r$ a consistent estimate of a basis of the space spanned by the β's can be obtained as follows. Choose $h = 2m$ and solve

$$det(\hat{A}_h - \lambda(\hat{A}_h + \frac{1}{T^2}\hat{A}_h^{-1})^{-1}) = 0$$

Let $\hat{H}$ be the matrix of eigenvectors corresponding to the r largest[10] eigenvalues of the above problem. Then $\hat{H} = (\beta_1, \ldots \beta_r)\Xi_r + O_p(\frac{1}{T})$ where $\Xi_r \in \mathbb{R}^{r \times r}$ with full rank. In other words the eigenvectors to the smallest eigenvalues form an estimate of the cointegrating space. Again the rate of approximation is $O_p(\frac{1}{T})$, i.e. the cointegrating space is estimated super-consistently.

3 Results of a simulation study

The aim of this section is to explore the finite sample behavior of the two discussed methods. The cited result of Wagner [18] shows that the Johansen procedure has some useful theoretical robustness properties. Since an analytical derivation of the limit distribution of the test statistics is not available under the discussed form of misspecification, it is unclear how much influence on the asymptotic distribution is really exerted by the MA polynomial $b(L)$.[11]
We want to study several aspects. For both tests we want to see whether the actual size of the test is approximating the nominal size. Since we are going to simulate ARMA systems, this serves as an indication of the empirical relevance of Theorem 2.1 in the case of the Johansen method. For the Bierens method, which is designed for ARMA processes, this gives an indication of the speed of convergence of the finite sample values of the test statistic to the tabulated

[8] The test is left-sided: the null is rejected if $\hat{\lambda}_{m-r,h}$ is smaller than a critical value. The critical values can be found in Table 2 in Bierens [3].
[9] See Table 1 in Bierens [2] for details.
[10] In the papers Bierens [2, 3] the eigenvectors to the smallest eigenvalues are incorrectly specified as the cointegrating vectors.
[11] The simulations have been performed using GAUSS 3.2, the programmes and further results are available from the author upon request.

(asymptotical) values.
Second, we want to analyze the behavior of the estimated cointegrating space and the distance, measured by the Hausdorff distance defined below, between the estimated and the true cointegrating space. This set-up allows us to distinguish the properties of the test from the properties of the estimation of the cointegrating space of the different methods. This allows for the detection of the weaknesses as well as the advantages of the different methods. As we shall see below the Johansen procedure has size distortions for some of the simulated systems for 50 observations. Nevertheless, the estimated cointegrating space, given the dimension, is estimated very precisely already for that sample size. This means that if prior information concerning the number of cointegrating relationships is available, e.g. from economic theory, small samples allow already for a quite precise estimation of the relationships by using the Johansen procedure. Another possibility that arises from this set-up is to combine several methods for testing and estimating for cointegration to exploit the advantages of the different methods. E.g. if one test has better size properties than another, but the estimates are more precise for the method with the test with larger size distortions, one could use the test result from the first method and the estimated cointegrating relationships from the second.[12]
Both of these aspects are investigated for different sample sizes to see whether the established consistency of the estimated cointegrated space of the Johansen procedure under misspecification is of empirical relevance and also to see the small sample properties of the Bierens procedure.[13]
As a distance measure between the estimated and the true cointegrating space we use, as indicated above, the Hausdorff distance, which is defined as follows: Let ζ and η be two subspaces of $\mathbb{R}^m$. The intersection of a subspace θ of $\mathbb{R}^m$ with the closed unit circle in $\mathbb{R}^m$ is denoted by $C(\theta)$,

$$C(\theta) = \{z \in \theta \,|\, \| z \| \leq 1\},$$

where $\| z \|$ is the Euclidean norm of z. Using this notation the distance d of ζ and η is given by the Hausdorff distance d_H of $C(\zeta)$ and $C(\eta)$, i.e.

$$d(\zeta,\eta) = d_H(C(\zeta), C(\eta)) = max(\rho(C(\zeta), C(\eta)), \rho(C(\eta), C(\zeta)))$$

where $\rho(C_1, C_2)$is given by

$$\rho(C_1, C_2) = \sup_{x \in C_1} \inf_{y \in C_2} \| x - y \| .$$

The first set of models that has been simulated are two-dimensional ARMA(2,1)

[12]In our case it turns out though that the Johansen procedure dominates the Bierens Procedure, for almost all ARMA systems considered. Further results are available upon request.

[13]A natural guess at this point is that for small samples, for which only low-order AR models can be estimated, Bierens' test is superior since it is designed for ARMA systems. Its non-parametric character on the other hand indicates that it might require bigger samples. For large sample sizes we would not expect to observe all too big differences. Things turn out to be different, however.

systems with one cointegrating vector adopted from Hargreaves [6]:[14]

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} u_{1t-1} \\ u_{2t-1} \end{bmatrix} +$$
$$+ \begin{bmatrix} -0.5 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1t-2} \\ u_{2t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} + \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \end{bmatrix} \qquad (3.1)$$

The parameter values for the MA polynomials that we have chosen are $\gamma_1 = -1$ and $\gamma_2 = -0.9$, systems with

$$\gamma_1 = \gamma_2 = -0.8, -0.5. - 0.2, 0, 0.2, 0.5, 0.8,$$

and $\gamma_1 = 1$ and $\gamma_2 = 0.8$.
The first system has a unit root in the MA polynomial, so that it does not satisfy the conditions of Theorem 2.1. Still, it is interesting to see the behavior of the Johansen estimates in the case of the presence of an MA unit root. The fifth system, the pure AR(2) system, serves as a point of reference. In the following discussion these systems will be referred to as MA1 to MA9.

The true cointegrating vector of the above system(s) is, suitably normed, given by $(1, -3)$, it is of course the space spanned by the second row of the matrix at the beginning of the first line of (3.1).

In the simulations presented below the order of the systems is assumed to be unknown. Since we are going to apply the Johansen procedure the order of an autoregressive approximation of the ARMA systems has to be selected. This will be done by selecting the lag length according to an information criterion. In the paper the results are given for choosing the lag length according to the AIC, the results are practically identical when the lag order is selected according to the BIC.
In Table 1 it can be seen that only for the systems with large positive autocorrelation of the ε_t's large lag lengths tend to be chosen. Also for the system with a unit root in the moving average polynomial a lag length of 2 is selected for an autoregressive approximation of the system.
Figures 1 and 2 show the probabilities for choosing the correct number of cointegrating vectors for the Johansen and the Bierens procedure.

Comparing the two figures one sees enormous differences in the performance of the tests. The Johansen procedure fails to show some reasonable performance only for the system with a unit root in the MA polynomial. This system violates our assumption at the top of Theorem 2.1, so some strange behavior had to be expected. For all the other systems, the nominal size tends to the asymptotical

[14] All the two- and three- dimensional systems have been simulated 5000 times. Time series of length 250 have been generated, the first 50 observations have been skipped. The time series of the different sample sizes have been constructed as the corresponding observations of the remaining 200 observations. The results related to the tests are all at the 5 % critical level. The results are almost unchanged for other significance levels and available from the author upon request.

	MA1	MA2	MA3	MA4	MA5	MA6	MA7	MA8	MA9
50	2	2	2	2	2	2	3	4	5
100	2	3	2	2	2	2	4	4	4
150	2	3	2	2	2	2	4	4	7
200	2	3	2	2	2	2	4	4	7

Table 1: Selected autoregressive order of an autoregressive approximation of systems (3.1) for different sample sizes using AIC.

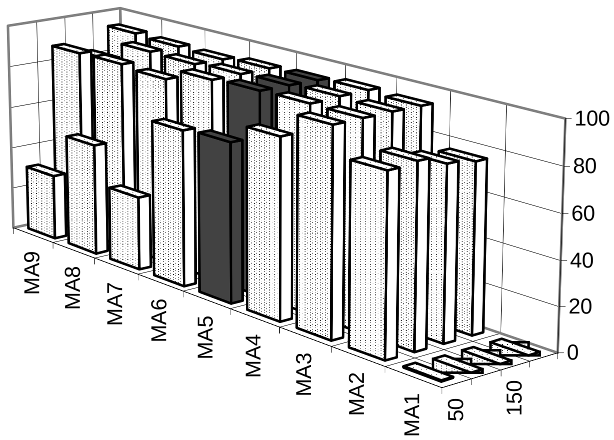

Figure 1: Acceptance probabilities of the correct number of cointegrating vectors for systems (3.1) using the Johansen trace test.

size.[15]
Only for the systems with a high positive autocorrelation in the MA polynomial[16] do we see a clear under-acceptance of the correct number of cointegrating vectors for a sample size of 50 observations. This effect becomes negligible for a sample size of 100 or larger.
Things turn out to be different for the Bierens procedure. Here reasonable results are only obtained for the systems with negative correlation in the MA polynomials, including the MA polynomial with a unit root. For all the other MA parts and all sample sizes the acceptance probability for choosing a cointegrating space of 1 remains below 10 %. Since the procedure is in principle constructed for ARMA systems, this observation raises some doubts about the usefulness of this procedure in empirical analysis. As we will see later, things are even worse for a set of three-dimensional systems.

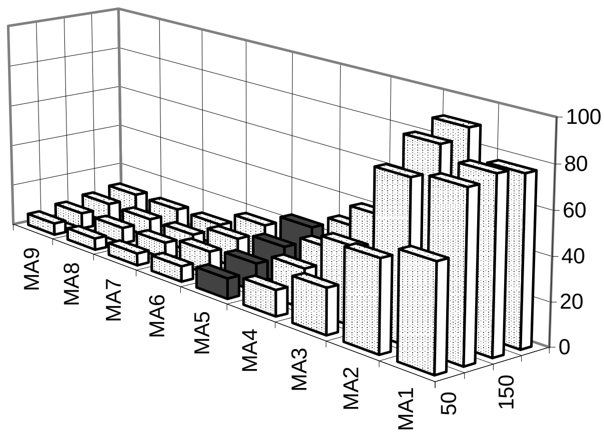

Figure 2: Acceptance probabilities of the correct number of cointegrating vectors for systems (3.1) using Bierens' non-parametric cointegration test.

Having seen that the performance of the procedures is quite different at the test step, we will now look at the distribution of the estimated cointegrating space, assuming the true dimension to be known. Since the true dimension is 1 and we are analyzing two-dimensional systems, after normalization only one element in the cointegrating vector is undetermined.[17] The empirical distribution of the second element is described by its mean, standard deviation, median, skewness and kurtosis.[18]

[15] The actual size tends to get close to the nominal since the acceptance probabilities are tending to values in the vicinity of 95 %, and the tests have been conducted at the 5 % critical level.

[16] These are the systems for which higher orders have been selected by AIC.

[17] Tables describing the features of the distributions for all sample sizes and all MA polynomial for both procedures are available upon request.

[18] The empirical distribution is calculated from taking the first solution vector of the eigen-

The general picture that emerges is the following. The (empirical) means are tending to the true value of -3 for increasing sample size for all systems and for both procedures. Generally, the values of the Johansen estimates are closer to the true value than the ones computed using Bierens' method. This is perfectly consistent with the observation that the standard deviations are smaller for Johansen's than for Bierens' method for all systems and all sample sizes. The same picture emerges for the median. Skewness and kurtosis are essentially of the same magnitude for both procedures.
In Figure 3 we see the Hausdorff distance of the vectors composed of the mean elements and the true cointegrating space. The four subpanels display the results for the four different sample sizes. Again, for all MA polynomials and all sample sizes, the Johansen procedure dominates the Bierens procedure. For the larger sample sizes the means of the Johansen estimates are practically identical to the true cointegrating space. Also, due to superconsistency, the mean of the Bierens' estimates is close to the true value, but not as much as Johansen's estimates.[19]
As a conclusion we find for the two-dimensional systems that in all the 'dimensions' that we have looked at the Johansen procedure strictly dominates the Bierens procedure, with the latter producing extremely bad results in the test step.

The second set of systems that has been simulated are three-dimensional ARMA(2,1) systems with a two-dimensional cointegrating space.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} u_{1t-1} \\ u_{2t-1} \\ u_{3t-1} \end{bmatrix} +$$
$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.7 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} \begin{bmatrix} u_{1t-2} \\ u_{2t-2} \\ u_{3t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix} + \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma 3 \end{bmatrix} \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \\ \epsilon_{3t-1} \end{bmatrix} \quad (3.2)$$

The MA polynomials used for simulation are $\gamma_1 = -1, \gamma_2 = \gamma_3 = -0.9$ and the following systems with identical entries

$$\gamma_1 = \gamma_2 = \gamma_3 = -0.5,\ 0,\ 0.6,\ 0.8$$

Again the first system, MA1, has a unit root in the MA polynomial. These systems will be referred to as MA1 to MA6 in the following tables and figures.

value problem in each repetition of the simulation. Our measure of skewness is

$$\frac{q_{97.5} - q_{50.0}}{q_{50.0} - q_{2.5}} - 1,$$

where q_i is the i-th quartile. As a measure of the kurtosis we use

$$\frac{q_{97.5} - q_{2.5}}{q_{99.5} - q_{0.5}} - \frac{1.96}{2.575}.$$

Both measures are equal to zero for a normally distributed random variable.

[19]The qualitatively same picture emerges for the median vector.

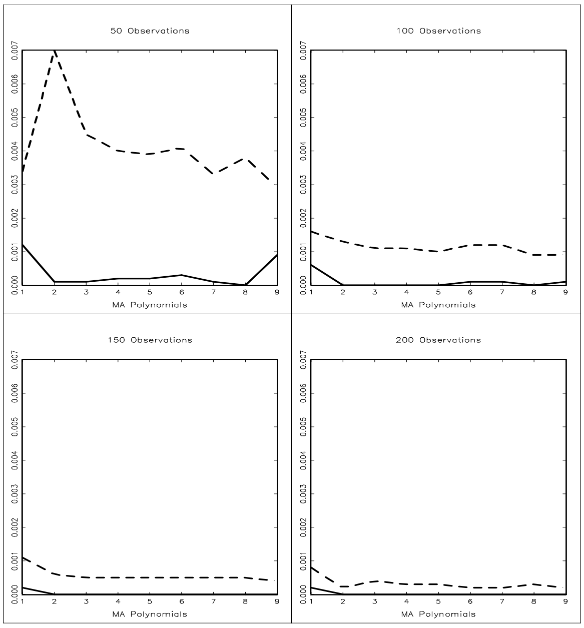

Figure 3: Hausdorff distances between the true and the mean of the estimated cointegrating spaces. The dashed line corresponds to Bierens' procedure, the solid line to Johansen's procedure.

	MA1	MA2	MA3	MA4	MA5	MA6
50	2	2	2	3	2	3
100	2	2	2	3	4	3
150	2	2	2	3	4	4
200	2	2	2	3	4	4

Table 2: Selected autoregressive order of an autoregressive approximation of systems (4.1) for different sample sizes using AIC.

The true cointegrating space is now two-dimensional, and a basis is given by

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

With a two-dimensional cointegrating space after normalization 4 elements are undetermined. We normalize the first element of the first vector to 1 and the second element of the second to 1.

Table 2 shows the chosen lag lengths for an autoregressive approximation of the different systems.

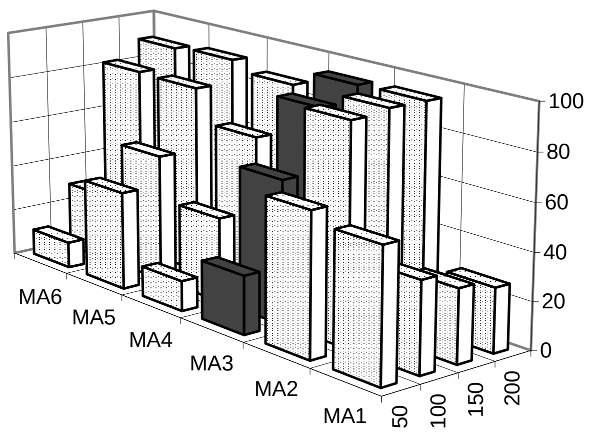

Figure 4: Acceptance probabilities of the correct number of cointegrating vectors for systems (3.2) using the Johansen trace test.

We start as before with looking at the acceptance probabilities for the correct dimension of the cointegrating space, which is 2 now, using both the Johansen and the Bierens test. For the Johansen method we see essentially the same picture as before. For samples of sizes 150 and 200 the acceptance probabilities tend

to values around 90 %. For smaller sample sizes the evidence is mixed. This was expected since the larger systems require the estimation of more parameters. On the other hand the picture for the Bierens procedure is quite surprising. Here for all systems but MA1, the system with a unit root in the MA polynomial, the acceptance probabilities for a two-dimensional cointegrating space are essentially 0. As can be seen from Figure 6, the Bierens procedure tends to accept no cointegration at all in most of the cases, with a frequency of about 80 to 90 %. This means that for systems as the ones used in the simulations the use of Bierens' cointegration procedure gives results that have to be taken with a great deal of caution.
For completeness' sake we will include the results of Bierens' method in the discussion of the distribution of the estimates of the cointegration space.[20] Again, the Johansen procedure outperforms the Bierens procedure substantially. E.g. the standard deviations of Johansen estimates are smaller than the standard deviations of Bierens estimates. For both procedures the standard deviations are larger than for the two-dimensional systems.

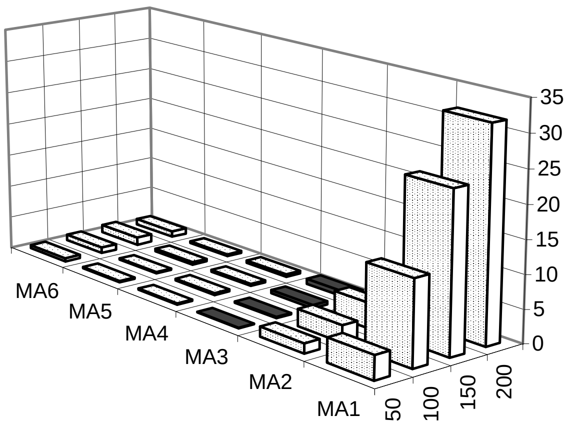

Figure 5: Acceptance probabilities of the correct number of cointegrating vectors for systems (3.2) using Bierens' non-parametric cointegration test.

The following Figure 7 shows the Hausdorff distance between the mean vectors and the true cointegrating space, again for all the sample sizes and the true number of cointegrating vectors. It can be clearly seen that the Johansen method is again achieving a good approximation of the true cointegrating space for all systems for 100 or more observations. The Bierens procedure only yields unsatisfactory results.
Now, since all the systems above have been artificial examples, we will look at some real data in the next section. This shall give some indication concerning

[20]Tables describing the features of the distributions can be obtained from the author upon request.

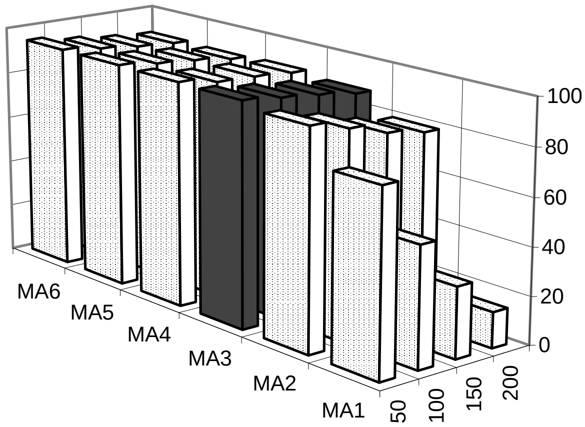

Figure 6: Acceptance probabilities of zero cointegrating vectors for systems (3.2) using Bierens' non-parametric cointegration test.

the relative performance of the two methods on actual data, especially whether Bierens' procedure achieves a more reasonable performance there.

4 An empirical example

In Figure 8 the data, quarterly consumption and output for Austria are displayed. The data are in real terms, seasonally adjusted and transformed to logs. The range is from 1976:IV to 1999:I.

The two series obviously exhibit very close co-movement, they are also from an economic point of view prime candidates for being cointegrated. As is well known, the permanent income hypothesis implies a cointegrating relationship between consumption and (permanent) income.
The first step of the analysis consists of testing whether the series can individually be regarded as being integrated of order one. The null hypotheses of random walk with drift cannot be rejected for both series for a battery of tests (like Augmented Dickey Fuller, Phillips-Perron, Bierens HOAC, Bierens NLADF). In conducting the Johansen procedure for cointegration the next decisions to be made refer to the choice of the lag length and the specification of the deterministic part. The deterministic component employed is an unrestricted vector of drifts. The optimal lag length, again chosen according to AIC, is 6.[21]

[21]BIC indicates an optimal lag length of 2. The results concerning cointegration are very similar to the model with 6 lags. The model with 2 lags suffers from some serial correlation of the residuals. The computations in this section have been performed using own GAUSS code, the package CATS in RATS and Bierens' EASYREG programme, which can be downloaded from Herman Bierens' home page. The results are the same across packages, so the surprising results of Bierens' procedure are not due to programming errors in the GAUSS programmes.

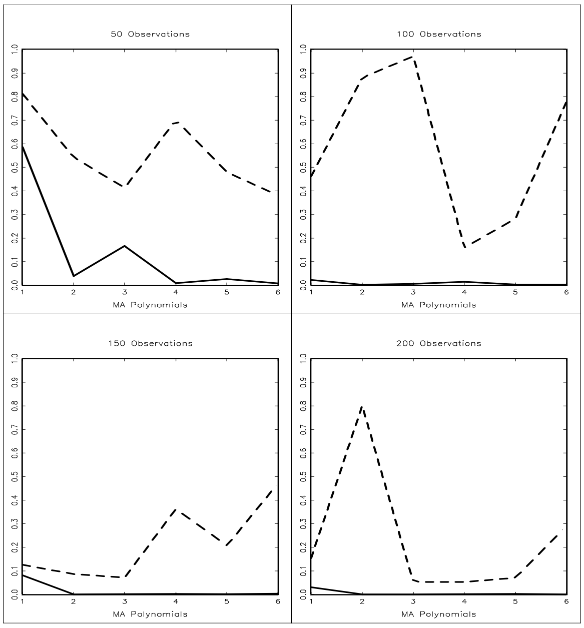

Figure 7: Hausdorff distances between the true and the mean of the estimated cointegration spaces. The dashed line corresponds to Bierens' procedure, the solid line to Johansen's procedure.

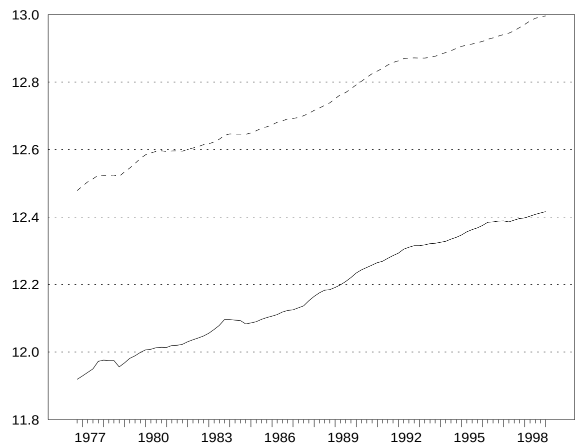

Figure 8: Austrian GDP (dotted line) and Austrian Private Consumption (solid line). The data are transformed to logarithms.

Eigenval.	ξ	η	$H_0 : r$	n-r	$\xi - 90\%$	$\eta - 90\%$
0.2221	21.1	21.2	0	2	10.60	13.31
0.0012	0.1	0.1	1	1	2.71	2.71

Table 3: The output of the Johansen cointegration analysis.

The output of the cointegration testing procedures is as follows. The Johansen procedure clearly indicates a one-dimensional cointegrating space (see Table 3). The estimated cointegrating vector is $(1, -1.024)$.[22] The corresponding stationary component, i.e. the disequilibrium, can be seen in Figure 9.

Although the second entry in this vector is very close to -1, the hypothesis of a unit income elasticity of consumption ($H_0 : \beta' = (1, -1)$) can be rejected.
The Bierens procedure gives a test result of a two-dimensional cointegrating space. This stands in clear contrast with both series being individually random walk with drifts.[23]
Taking the first solution vector of the Bierens eigenvector procedure to see the cointegrating vector estimated by this procedure, gives the vector $(1, -0.91)$. With the corresponding hypothesis test the hypothesis of a unit elasticity cannot be rejected here. Although -0.91 is further away from -1 than -1.024, the null hypothesis cannot be rejected now, contrary to the Johansen hypothesis test.

The data and detailed results of the estimation procedures are available upon request.

[22]The ordering of the variables in the estimated VAR is log-consumption and log-output.

[23]Bierens [2, 3] also describes a method to estimate the dimension of the cointegrating space instead of testing for it. In the present example also the estimated dimension of the cointegrating space is equal to 2.

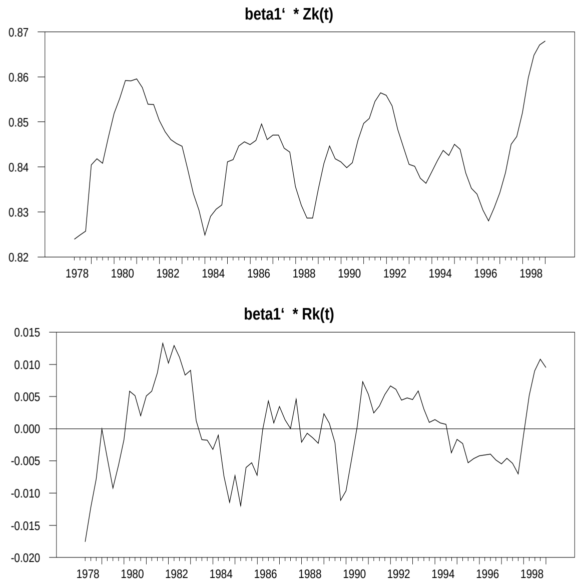

Figure 9: The cointegrating relationship estimated by the Johansen method. The upper panel displays the actual disequilibrium ($\beta' x_t$ in the notation of Section 2) and the lower panel displays the disequilibrium corrected for short-run effects ($\beta' R_{pt}$, using again the notation of Section 2).

This fits to our observations made in Section 3, where we have seen that the standard deviations of the Bierens estimates are larger than those obtained by using the Johansen method.
Thus, our example also does not really provide support for the method presented by Bierens.

5 Conclusions

The above results do not provide much support for using the Bierens cointegration procedure. One might argue that the systems analyzed in the simulation studies are probably too well described by low-order autoregressive systems, so that the Johansen method is working well in these cases. Still, the Bierens method should as well be reasonably applicable in these cases. But what we have seen cannot be regarded as promising results.[24] Another possible reason for the bad performance of the Bierens procedure could be related to the fact that we have used only samples up to 200 observations. However, since cointegration has found most of its applications in macroeconomics, these are the relevant sample sizes.[25] As we have seen the main drawback of the Bierens procedure is the power of the tests, maybe also the critical values could be re-examined.
A conclusion that can be drawn from this comparison is that the Johansen procedure on the other hand is remarkably robust already for small sample sizes.[26] Taking also into account, as mentioned in the introduction, that the Johansen-VAR framework allows for a wide variety of hypotheses to be tested, as well as an extension to the I(2) case, it seems to be the method that should be used preferably, at least until some further understanding concerning the properties of the method developed by Bierens has been achieved.
One thing that remains to be done is to analyze in more detail conditions under which the Bierens procedure is delivering results that are in terms of their quality comparable to the ones obtained by Johansen or other 'standard' methods. For the time being the question raised in the title of this paper can only be answered as follows: The two methods are up to now neither complements nor substitutes, but[27] the Johansen method dominates the method proposed by Bierens. A conclusion that can be drawn from this result is that it is necessary to gain further understanding of the Bierens procedure since there is the need to have methods for estimation and testing in cointegrated systems also in such cases when the systems are not pure autoregressive systems. Having reliable

[24]As discussed in Section 2, the Bierens procedure is applicable to ARMA systems integrated of order 1, so for the relatively simple systems analyzed above the results could have been expected to be better.

[25]Furthermore, in the figures there is no tendency for an improvement with growing sample sizes visible.

[26]One point that should be regarded with caution however is that we have ignored deterministic parts in the simulation. But the most plausible candidates for deterministic terms, like unconstrained drift or trend, are concentrated out in the first steps of the procedure anyway, so from this point of view not too much changes should be expected.

[27]For the systems and data set analyzed, to be careful.

methods for this problem at hand, one can at least validate the results gained e.g. in a VAR study using the Johansen procedure.

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